

Dear Referee,

thank you very much for your comments and please accept my apologies for the long delay in producing the corrected version of the paper. As you will see the paper was almost completely re-written in order to provide it with detailed proofs as well as to connect it to the new results that have appeared since.

The new paper does not cover all of the material of the previous one. It will be followed (possibly in a different journal) by another one with the preliminary title “Generalized term C-systems” where the results of this paper will be specialized to the case of pairs (R, LM) arising from two-sorted binding signatures that lie in the foundation of the syntax of type theories (with the two sorts being the sort of type expressions and the sort of element expressions).

In this paper I concentrate on the results related to general pairs (R, LM) . In particular, Lemma 2.2 of the previous version that has attracted one of your comments is not a part of the new one. Nevertheless, I try below to reply to all of your comments whether they apply to the new version or not.

1. Following your suggestion the “Introduction” section was expanded substantially and should better explain the place of the present paper in the general scheme of my approach to the dependent type theory.
2. Comment to page 1. The subject of the sentence as I see it is ”A modified axiomatics of C-systems *and* the construction of new C-systems as sub-objects and regular quotients of the existing ones” which I think requires a plural form of “is”.
3. Comment to page 2. This does not seem to have survived in the new introduction.
4. Comments to page 3:

- (a) In the notation $R_{A,1}$, “1” was to underlie that one considers the projection to the first factor (\mathcal{C}) . One can also consider the same construction with an object B of \mathcal{C} and $pr_{\mathcal{D}}$ which would then be denoted by $R_{B,2}$.
- (b) The misunderstanding is due to my mistake. In the formula for $bind(f)$ one should consider $\eta(X', A)$. Then:

$$\begin{aligned} dom(f, pr_{\mathcal{D}}(\eta(X', A))) &= (dom(f), dom(pr_{\mathcal{D}}(\eta(X', A)))) = (X, pr_{\mathcal{D}}(dom(\eta(X', A)))) = \\ &= (X, pr_{\mathcal{D}}(X', A)) = (X, A) \end{aligned}$$

and

$$\begin{aligned} codom(f, pr_{\mathcal{D}}(\eta(X', A))) &= (codom(f), codom(pr_{\mathcal{D}}(\eta(X', A)))) = \\ &= (pr_{\mathcal{C}}(R(X', A)), pr_{\mathcal{D}}(codom(\eta(X', A)))) = (pr_{\mathcal{C}}(R(X', A)), pr_{\mathcal{D}}(R(X', A))) = R(X', A) \end{aligned}$$

Therefore, $bind(f, pr_{\mathcal{D}}(\eta(X', A)))$ is defined and

$$\begin{aligned} dom(pr_{\mathcal{C}}(bind(f, pr_{\mathcal{D}}(\eta(X', A)))))) &= pr_{\mathcal{C}}(dom(bind(f, pr_{\mathcal{D}}(\eta(X', A)))))) = \\ &= pr_{\mathcal{C}}(R(X, A)) \\ codom(pr_{\mathcal{C}}(bind(f, pr_{\mathcal{D}}(\eta(X', A)))))) &= pr_{\mathcal{C}}(codom(bind(f, pr_{\mathcal{D}}(\eta(X', A)))))) = \\ &= pr_{\mathcal{C}}(R(X', A)) \end{aligned}$$

Generalization to relative monads and careful verification of the axioms of a relative monad will be included to the “Generalized term C-systems” paper.

5. Comment to pp. 3-4. The names “left module” and “right module” are as in the papers by Hirschowitz and Maggesi where the concept of a module over a monad is introduced and studied. I only followed their naming. I am not sure what do you call Definition 3.1 of [6]. There is Definition 7 in Section 3.1 (p. 222) of [6] where such objects are defined and called left modules.

About one-letter names as opposed to multi-letter ones. Paucity of one-letter names pushes one to avoid introducing notations for some objects and ultimately leads to vague exposition. Multi-letter names while they look strange at first allow one to be more precise.

6. Comment to p.19. Thank you, this remark was wrong and is not in the paper anymore.

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